The changing concept of matter in H. Weyl's thought, 1918 – 1930

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Abstract

During the "long decade" of transformation of mathematical physics between 1915 and 1930, H. Weyl interacted with physics in two highly productive phases and contributed to it, among others, by his widely read book on *Space - Time - Matter (Raum Zeit - Materie)* (1918 – 1923) and on *Group Theory and Quantum Mechanics (Gruppentheorie und Quantenmechanik)* (1928 - 1931). In this time Weyl's understanding of the constitution of matter and its mathematical description changed considerably. At the beginning of the period he started from a "dynamistic" and geometrical conception of matter, following and extending the Mie-Hilbert approch, which he gave up during the year 1920. After transitional experiments with a singularity (and in this sense topological) approach in 1921/22, he developed an open perspective of what he called an "agency theory" of matter. The idea for it was formulated already before the advent of the "new" quantum mechanics in 1925/26. It turned out to be well suited to be taken over to the quantum view as a kind of "heritage" from the first half of the decade. At the end of the period, Weyl completely renounced his earlier belief in the possibility to "construct matter" from a geometrically unified field theory. He now posed the possibility of a geometrization of the mathematical forms underlying the rising quantum physical description of matter as a completely open problem for future research.

Introduction

It may appear a strange question to ask for the changing views of a mathematician on the concept of matter. Why not pose it for a natural scientist or a philosopher? But Hermann Weyl was, as we know, a bit of all of them. His views on mathematics and their foundations made it impossible to separate mathematics and its "meaning" from broader contexts of its use as a conceptual form and as a symbolic tool for the understanding of nature (or at least some aspects of it).

During the "long decade" of transformation of mathematical physics, as we may call the time between 1915 and 1930, with the rise of the general theory of relativity (GRT) and the origin of the "new" quantum mechanics (QM), H. Weyl interacted with physics in two highly productive phases and contributed to the development of both, theoretical physics and the mathematical concepts and methods in it. The first phase lasted from 1916 to 1923 and had as main outspring his widely read book on *Space - Time - Matter (Raum Zeit - Materie)* (Weyl 1918*b*), which we will also refer to by RZM. In

this period the book had five successive editions with considerable extensions and/or alterations well documenting the shifts in the understanding of the subject by its author. Some of these changes were of a more technical nature for general relativity or the mathematics involved, others were of more basic nature, including in particular the changing characterization and mathematical description of matter. In the middle of the 1920s Weyl worked on the representation theory of Lie groups (1924/1925) and wrote a book on the philosophy of mathematics and the natural sciences (1925/1926), before he started to contribute actively to the rising quantum mechanics, culminating in his second book about mathematical physics, *Group Theory and Quantum Mechanics* (Weyl 1928).

The growing awareness of the unreducible and far-reaching role of quantum properties had already contributed to considerable shifts of Weyl's concept of matter during the first phase of involvement in physics. In the second half of the "long decade" his views were deeply transformed by the rise of quantum physics. This transformation was, of course, much more than a personal experience. It reflected the experience of the whole community of researchers in basic physics of the time, although seen from a specific Weylian perspective. As such, it may be illuminating for a historical and philosophical understanding of the transformation of the concept of matter, brought about by the tension resulting from the unfinished "double revolution" of GRT and QM during the 1920s.[1] In spite of the drastic difference between Weyl's concept of matter at the beginning of the period and at the end of it, we easily perceive a common thread linking both ends. This common underlying feature is a dynamistic view of matter. This characterization has to be understood in a general, philosophico-conceptual sense which *may* be related, but *need not* be, to the electrodynamical picture of matter which gave a new thrive to dynamism among physicists and mathematicians of the early 20th century.

In the history and philosophy of physics, the dynamistic view of matter in the early 20th century is often restricted to the exclusively electromagnetic approach. Such a restriction shadows off the intricate link to the quantum theoretical phase, which played a role for some of the protagonists of the period. Of course, also Weyl started from Mie's electrodynamical theory of matter when he first looked for an adequate "modern" mathematical expression of such a dynamistic view. From this basis he developed his program of a geometrically unified field theory in the first phase of his involvement in mathematical physics.[2] The impact of quantum physics replaced classical

[1] A. Pais' description of the change of matter concepts by the rising quantum theory as "the end of the game of pebbles" (Pais 1986, 324) fits already well to this shift, although Pais used it as a header for the rise of second quantized fields starting in the late 1920s.

[2] For Weyl's first phase of involvement in mathematical physics compare (Sigurdsson 1991, Scholz 2001*b*), for broader views on unified field theories see (Vizgin 1994, Goldstein/Ritter 2000, Cao 1997, Goenner 2004).

field pictures by quantum stochastical descriptions of the "agency nature" of matter, as Weyl liked to call it. At the end of the period discussed here, he completely renounced his earlier belief in the possibility to "construct matter" from a geometrically unified field theory. He now posed the possibility of a geometrization of the mathematical forms underlying the rising quantum physical description of matter as a completely open problem for future research.

In this article I present a kind of "logitudinal section" through the long decade, oberved along the trajectory of a single person, who was partially a contributor and partially a well informed observer of the development.[3] We start with Weyl's turn towards Mie's theory of matter, his own contribution to it, and his rather early distachment, which was related to the influences of early quantum mechnics without being a necessary conclusion from it. After a short phase of relaxation of classical explanations of matter, by a combination of metrical and topological aspects (matter characterized by singularities in space-time), Weyl developed an open perspective of what he called an "agency theory" of matter. The idea for it was formulated already before the advent of the "new" quantum mechanics in 1925/26. It turned out to be well suited to be taken over to the quantum view as a kind of "heritage" from the first half of the decade.

Adherence to Mie's dynamistic approach to matter

After Weyl came back to neutral Switzerland from his war duty in the German army in May 1916, he started a completely new phase of his research, which was imbued by a longing for a sounder basis of knowledge.[4] For him, this meant to work in a broad and interconnected set of fields comprising the foundations of analysis, differential geometry, general relativity, unified field theory and the basic structures of matter. Only if we take this broad range of intellectual activities into account, we can get an adequate sense of Weyl's conceptual and theoretical moves inside the single fields. Let us have a look at some points of such interconnections:

- In the foundations of mathematics our author shifted from his own constructive-arithmetical approach for a characterization of the concept of continuum (Weyl 1918*a*) to a kind of Brouwerian intutionism (Weyl 1921*b*). For a while, he believed Brouwer's approach to possess an intimate connection to his ideas in purely infinitesimal geometry. Weyl could well characterize "purely infinitesimal" structures on the level of differential geometry by his generalization of a Riemannian metric by combining a conformal structure with a *length connection*

[3]A complementary view at several "transversal" sections (in time) with a broad evaluation of authors and approaches is presented in (Goldstein/Ritter 2000).

[4]See (Sigurdsson 1991, 64ff.), (Schappacher 2003).

$\varphi = \sum_i \varphi_i dx^i$. It was, however, much more difficult to give them a mathematical meaning on the foundational (and topological) level. Here a precise conceptual characterization was lacking. Weyl was well aware of this deficiency which contributed to tensions and shifts inside his foundational contributions. For a while, Brouwer's "revolutionary" approach to the continuum (as Weyl called it in 1920) appeared him to offer a promising road.[5]

- For some years, Weyl considered his gauge geometrical generalization of the Riemannian metric as the proper approach for a unified field theory of gravitation and electromagnetism and, moreover, a field theory of matter based upon it.[6]

- Rising doubts with respect to the physical feasibility of this immediate physical interpretation of gauge geometry contributed to a turn towards a more basic philosophico-conceptual analysis of the principles of congruence geometry in Weyl's *mathematical analysis of the problem of space*.[7]

- The necessity, or at least usefulness, to accept classical logical prinicples (excluded middle) in the proof of the main theorem of the analysis of the space problem contributed to rethink his radical position in the foundations of mathematics.

In the second point indicated above, the field theoretic approach to matter constitution, Weyl was deeply influenced by Mie's electromagnetic theory of matter which he got to know through Hilbert's modification during the autumn 1915.[8] Hilbert attempted to arrive at a kind of mathematical synthesis of Mie's and Einstein's ideas on electromagnetism (Mie) and gravitation (Einstein). He indicated how to find a united Hamiltonian for gravitation and electromagnetism in a generally covariant setting.[9] He was convinced, that in such a classical united field theory the riddles of the grainy structure of matter should be solvable. H. Weyl and F. Klein were not convinced that Hilbert's attempted "synthesis" of Mie and Einstein was acceptable as a physical theory. They argued for a broader understanding of Hilbert's approach. E. Noether's mathematical analysis of Hilbert's invariance conjectures (later "Noether theorems") contributed an essential mathematical stepping stone for it.[10]

[5]Compare (Hesseling 2003, 121ff.), (Scholz 2000).

[6]See footnote 2.

[7](Scholz 2004*a*)

[8]Compare (Corry 1999*a*, Corry 1999*b*, Kohl 2002, Sauer 1999, Vizgin 1994).

[9]For a discussion of Hilbert's research program building upon and extending Mie's field theoretic matter theory see (Sauer 1999), for a critical evaluation of Hilbert's relation to Einstein's theory of general relativity (Corry 1997, Renn 1999).

[10]See (Rowe 1999, Brading 2002).

Although a central point of the scepticism resulted from the unclear role of energy conservation in Hilbert's approach, Weyl was, moreover, not convinced that Hilbert's approach was able to lead to a unification of gravitation and electromagnetism, in which matter structures were better derivable than in Mie's original version. In the first edition of his book he therefore discussed a field theoretic matter concept essentially as in Mie's original purely electromagnetic approach (Weyl 1918*b*, §25). Hilbert's generalization was only mentioned in passing, in the section which treated the modification of the Hamiltonian principle for electromagnetism by gravitation (Weyl 1918*b*, §32). On the other hand, Weyl presented Mie's approach in such a convinced rhetoric form that the reader might easily get the impression that Mie's research goal had already nearly been achieved. The desired result (derivation of a "grainy" structure from field laws) seemed close to sure. After a comparison of Mie's theory with Maxwell-Lorentz's, Weyl stated:

> The theory of Maxwell and Lorentz cannot hold for the interior of the electron; therefore, from the point of view of the ordinary theory of the electrons we must treat the electrons as something given *a priori*, as a foreing body to the field. A more general theory of electrodynamics has been proposed by *Mie*, by which it seems possible to derive the matter from the field ....(Weyl 1918*b*, 165)

This formulation was kept unchanged by Weyl in the next three editions.[11] He only changed it during the last revision for the fifth edition (1923). Then he clearly expressed the open status of Mie's attempt and presented it, in an essentially didactical approach, as nothing but an *example* of a physical theory "which agrees completely with the recent ideas about matter" (Weyl 1918*b*, ^{5}1923, 210).

Mie's proposal fitted beautifully to Einstein's detection of the energy-mass equivalence of special relativity and seemed to extend it. In a passage commenting the equivalence $E = mc^2$, Weyl argued:

> We have thus attained a new, purely dynamical view of matter (footnote: Even Kant in his "Metaphysische Anfangsgründe der Naturwissenschaft" teaches the doctrine that matter fills space not by its mere existence but in virtue of the repulsive forces of all its parts.) Just as the theory of relativity theory has taught us to reject the belief that we can recognize one and the same point in space at different times, *so now we see that there is no longer a meaning in speaking of the same position of matter at different times.* (Weyl 1918*b*, 162), (Weyl 1922, 202)

[11]It thus appears verbally unchanged in the third edition on which H.L. Brose's English translation is based (Weyl 1922, 206). Here, as in other cases, our English quotes from RZM are following Brose's translation, where available.

Already here, in the context of special relativity, he described an electron as a kind of "energy knot" which "propagates through empty space like a water wave across the sea", and which could no longer be considered as element of some self-identical substance. Then, of course, there arose the problem to understand both, this kind of propagation of energy, and the stability of the "energy knot". Weyl stated the new challenge of (special) relativity to field theory, which arose from a dynamical understanding of matter/energy:

> The theory of fields has to explain why the field is granular in structure and why these energy-knots preserve themselves permanently from energy and momentum in their passage to and fro (...); therein lies the *problem of matter*. (Weyl 1918*b*, 162, emphasis in original) (Weyl 1922, 203)

Like the dynamists of the early 19th century, Weyl now insisted that atoms could not be considered as invariant fundamental constituents of matter:

> *Atoms and electrons are not*, of course, *ultimate invariable elements*, which natural forces seize from without, pushing them hither and thither, but they are themselves distributed continuously und subject to minute changes of a fluid character in their smallest pieces. It is not the field that requires matter as its carrier in order to be able to exist itself, but *matter* is, on the contrary, *an offspring of the field.* (ibid.).[12]

It seems worthwhile to remark that these general passages on the dynamistic outlook on the problem of matter were *not changed* by Weyl until (and including) the fifth edition of his book in 1923. On the other hand, the special role attributed to Mie's theory, or to his own unified field theoretic approach, underwent considerable changes during the following years. But in spite of all his enthusiasm for the new role of field theory in the understanding of matter, Weyl indicated already in 1918 after his presentation of Mie's theory, that something new was rising at the (epistemic) horizon, which might have unforeseen consequences in the future. He compared the actual status of field physics with the seemingly all-embracing character of Newtonian mass-point dynamics in the Laplace program at the turn to the 19th century and warned:

> Physics, this time as a physics of fields, is again pursuing the object of reducing the totality of natural phenomena to *a single physical law:* it was believed that this goal was almost within reach when the mechanical physics of mass points, founded upon Newton's Prinicipia, was celebrating its triumphs. But also today, provision is taken that our trees do not grow up to the sky.

[12] Translation slightly adapted, E.S.

> We do not yet know whether the state quantities underlying Mie's theory suffice for a characterization of matter, whether it is in fact purely "electrical" in nature. Above all, the dark cloud of all those appearances that we are provisionally seeking to deal with by the quantum of action throws its shadow upon the land of of physical knowledge, threatening no one knows what new revolution. (Weyl 1918*b*, 170)[13]

A geometrical extension of Mie's theory of matter

A few months after his book manuscript was finished, Weyl developed his concept of a generalizated *Weylian metric* on a differentiable manifold. In technical terms his metric was given, and still can be characterized, by an equivalence class of pairs, $[(g, \varphi)]$, consisting of a (semi)Riemannian metrics $g = \sum_{i,j} g_{ij} dx_i dx_j$ and a differential form ("length connection") $\varphi = \sum_i \varphi_i dx_i$, up to equivalence by conformal factors in the Riemannian component of the metric and "gauge transformation" of the length connection form.[14] This generalization allowed a seemingly natural interpretation of the potential of the electromagnetic field by the length connection and thus a metrical unification of the main physical fields known at the time, gravitation (g) and electromagnetism (φ). Weyl considered this structure as an important step forward for the Mie program of a dynamical characterization of matter. He published about it in several articles, starting in 1918. In the following year he included the approach into the third edition of his book (Weyl 1918*b*, ^{3}1919).

The first edition had ended with a section on cosmology, "Considerations of the world as a whole" (Weyl 1918*b*, §33). In the third edition two new sections were added, one on "the world metrics as the origin of the electromagnetic phenomena", containing an introduction to Weyl's unified field theory, and one on "matter, mechanics and the presumable (mutmaßliches) law of the world", in which Weyl's extension of the Mie program was sketched. Like in the first edition, Hilbert's extension of Mie's program was only indirectly mentioned in the section on the combined Hamiltonian principle of electromagnetism and gravitation. On the other hand, the last section culminated in Weyl's own attempt to overbid both Mie and Hilbert by a derivation of the discrete "granular" matter structures from his gauge invariant action principle. In his lecture course on mathematics and the knowledge of nature of the winter semester 1919/20 , Hilbert countered by an acid remark that such a perspective would lead to a kind of "Hegelian physics", in which the "whole world process would not go becond the limited content of a finite thought" (Hilbert 1992, 100). He did not explain, though, why this kind of analysis

[13] Unchanged in all editions, last one in (Weyl 1918*b*, ^{5}1923, 216). Translation from (Weyl 1922, 212) slightly adapted by E.S.

[14] (Varadarajan 2003, Vizgin 1994)

should not apply to his own program just as well .

In 1919 Weyl was at the high point of enthusiasm for his new theory. The new section in the third edition of his book started with a rhetoric trumpet-blast:

> We rise to a final synthesis ... (Weyl 1918*b*, ^{3}1919, 242)[15]

Part of his enthusiasm resulted apparently from the realization that gauge invariance with respect to the change of the length gauge led to a new invariance principle, which in Weyl's semantics of the approach could only be the invariance of electrical charge (Weyl 1918*b*, ^{3}1919) (Weyl 1922, 293). That was, of course, a great achievement of lasting importance, even if the specific version of gauge invariance had later to be given up.[16] But Weyl hoped for more. He expected that on the one hand the cosmological modification of the Einstein equation should be a natural result from his gauge geometry. On the other hand the stable solutions of the equations for the "problem of matter", satisfying adequate regularity conditions should lead to a discrete set of solutions depending on some parameter β. This expectation had a (formal) similarity to a set of "discrete eigenvalues" of an operator, although here the operator was not linear.

The problem was, in fact, characterized by a non-linear differential equation of great complexity. Even Weyl guessed that the available tools of analysis would probably neither suffice for a proof of their existence, nor for an approximative calculation (Weyl 1918*b*, ^{3}1919, 260). This remark made the epistemic status of Weyl's "discrete solutions" highly problematic. It turned them rather into a symbol for a natural philosophical speculation than into an object for research in mathematical physics. Weyl continued the discussion by a beautiful remark.

> The corspusculae which correspond to the possible eigenvalues had to coexist in the same world besides each other or in another, mutually enforcing on another subtle modifications of their intrinsic structure; strange consequences seem here to arise for the organization of the universe; perhaps they may make comprehensible its stillness in the large and unrest in the small. (Weyl 1918*b*, ^{3}1919, 261)

When Weyl wrote these lines, he was at the peak of his belief in a strong unification program of forces and matter, which could be constructed on purely geometrical grounds. In the last long passage of the new added

[15]"Wir erheben uns zu einer letzten Synthese." Brose's translation reduced the kick of enthusiasm considerably: "We now aim at a final synthesis" (Weyl 1922, 282). Weyl did not weaken the rhetoric until and including the fifth edition, although he slightly revised its wording by adding a "nun (now)" (Weyl 1918*b*, ^{5}1923).

[16]See (Vizgin 1994, Brading 2002) and for a detailed, historically oriented discussion of the underlying mathematics (Varadarajan 2003).

sections we find a discussion of how he now saw the relationship between geometry and physics in the light of his recent findings.

> We have realized that physics and geometry coincide with each other and that the world metrics is one, and even the only one, physical reality. Thus, in the final consequence, this physical reality appears as nothing but a pure form; geometry has not been physicalized but physics has been geometrized (nicht die Geometrie ist zur Physik, sondern die Physik ist zur Geometrie geworden). (Weyl 1918*b*, ^{3}1919)

H. Weyl was now at the apogee of the belief in a strong unification which was both, deeply reductionist and highly idealistic. In his eyes, physics seemed to be transformed to a purely formal status and was absorbed by geometry. Matter had seemingly become an epiphenomenon of the "world metrics" which started to acquire a slightly mystical flavour. In the mind of our protagonist, the physicalizing tendency of geometry among leading protagonists of the 19th century, including researchers like C.F. Gauss, N.I. Lobachevsky, and B. Riemann, appeared turned upside down — even though in this extreme form for only a year or two.

As we will see in a moment, this conviction did not hold for long. Already in the fourth edition, this extremely reductionist passage was cancelled by its author. Now the book ended with another, less reductive passage on the unifying power of the mind and an éloge of the "chords from that harmony of the spheres of which Pythagoras and Kepler once dreamed" (Weyl 1922, 312). Weyl did not hide that he had changed his mind; in a separate article written for the physical community shortly after the revisions for the fourth edition, he explained frankly:

> From the first edition of RZM to the third one I took the position of (...) [a purely field theoretic characterization of matter, E.S.], as I was charmed by the beauty and unity of pure field theory; in the fourth edition, however, I lost confidence in the field theory of matter by striking reasons and changed to the second point of view [of a primacy of matter, irreducible to interaction fields, E.S.] . (Weyl 1921*a*, 242)

Let us therefore have a look at the "striking resons" for this ontological shift at the beginning of the 1920s.

From speculations on the "the causal and the statistical view of physics" to a break with Mie's theory of matter

In the year 1919 Weyl gave a talk to the Swiss *Naturforschende Gesellschaft* on the relationship between the causal to the statistical view of physics which

was published a year later (Weyl 1920). This paper has been strongly criticized by P. Forman in his otherwise very stimulating article on Weimar culture and its influence on the discourse among physicists (Forman 1971) as a document of an "antirational" kind of "conversion to acausality". We need not take up here again the broader debate on the question how "antirational" the move was and what kind of "acausality" was at stake here.[17] It may suffice to add that for Weyl the topic of his talk contained a challenging combination of questions in the conceptual foundations of contemporary physics, including the rising "clouds" of quantum phenomena, with the question of how modern natural science can be made compatible with metaphysical considerations of the existential experience of the openness of evolving life processes and of the freedom of personal actions. A central topic of this talk was the directedness and irreversibility of time, which appeared Weyl to be linked to some process level of irreducibly statistical nature.[18]

The talk took place about the time of his turn to Brouwer's intuitionism. In *this respect* we could even speak of a kind of "conversion".[19] Weyl speculated that perhaps Brouwer's approach could lead to a solution of several fundamental problems at one strike. In mathematics he hoped for an answer to the foundational question of the concept of the mathematical continuum and for a philosophically and mathematically sound characterization of the topological "substrate" of purely infinitesimal geometry; in physics he expected a break with the rigidity of the causality structure in classical mechanics ("Gesetzesphysik") and an access to understand the irreducible directness of time. He expected that satisfying answers to all these questions might have some intimate link to the open process of "becoming" inherent in Brouwer's choice sequence for the characterization of the intuitionistic continuum:

> Finally and foremost, it is inbuilt into the essence of the continuum that it cannot be treated as a rigid being, but rather only as something what is continuously evolving in an infinite, inward bound process of becoming. (Weyl 1920, 121)

In this speculative thought, Weyl hoped to find a common thread binding together the foundations of analysis, "purely infinitesimal" geometry, the directedness of time flow in the physical world, its determinative openness, and a conjectured irreducibly stochastical nature of physical laws, which would break with the classical kind of lawfulness ("causality" in the language of the time).

[17]See the illuminating comments and critique of Forman's original presentation in (Hendry 1984, Sigurdsson 1991, Stöltzner 2002) and also the modifications in (Forman 1980).

[18]Weyl hinted at the possibility that the classical mechanical discussion on ergodicity had to be revised the light of "some mysterious discontinuity" introduced recently by quantum theory (Weyl 1920, 118).

[19]Compare, e.g., (Hesseling 2003, 127).

In September 1920, during the discussions of the Bad Nauheim meeting of the German *Naturforscher Versammlung* and from a draft manuscript of Pauli's contribution on relativity to the *Enzyklopädie der Wissenschaften* Weyl got to know content and reason of Pauli's critical evaluation of his modified version of the Mie theory of matter. This conjunction of detailed scientific criticism, coming from a personally close, young expert in the field, with his own most recent conceptual and metaphysical speculations, undermining the classically deterministic field structures anyhow, led Weyl to give up the belief in his program of a geometrically unified field theoretical derivation of matter structures. At the end of the year, in a letter to Felix Klein, in which he reported on his recent advances on mathematical and physical questions (included or not into the just finished fourth edition of RZM), he reported among others:

> Finally I thoroughly distached myself from Mie's theory and came to a different position with respect to the problem of matter. I no longer accept field physics as the key to reality. The field, the ether, appears to me only as a *transmitter* of effects, which is completely feeble by itself; while matter is a reality lying beyond the field and causing its states .... (*Letter H. Weyl to F. Klein, December 28, 1920* 1920)[20]

Similar phrases are to be found close to the end of the fourth edition of RZM. Here the last section, containg Weyl's version of Mie's theory was no longer announced under the emphatic title "Matter, mechanics, and presumable world law" as in the third edition. The discussion was now only presented as a "development of the simplest principle of action ...".

It contained a short discussion of some consequences of Weyl's gauge invariant quadratic action $S^2\sqrt{|detg|}$ for the Hamiltonian of a combined theory of gravitation and electromagnetism, with S the scalar curvature of Weyl geometry. Now he commented that this action is only the "simplest assumption for calculation", for which the author no longer wanted to "insist that it is realised in nature" (Weyl 1922, 295). For anybody who continued to read the book until the end, Weyl made clear that he now conjectured a close interrelation between the directedness of time flow with quantum jumps as seen in the Bohr model of the atom. That was no longer compatible with the classical structures of time invertible determinism:

[20]"Endlich habe ich mich gründlich von der Mie'schen Theorie losgemacht und bin zu einer anderen Stellung zum Problem der Materie gelangt. Die Feldphysik erscheint mir keineswegs mehr als der Schlüssel zu der Wirklichkeit; sondern das Feld, der Äther ist mir nur noch der in sich selbst völlig kraftlose *Übermittler* der Wirkungen, die Materie aber eine jenseits des Feldes liegende und dessen Zusände verursachende Realität. Mit dem "Weltgesetz" (Hamiltonsches Prinzip), das die Wirkungsübertragung im Äther regelt, wäre noch gar wenig für das Verständnis aller Naturerscheinungen gewonnen." (ibid. emphasis in original); compare (Sigurdsson 1991).

> We must here state in unmistakable language that physics at its present stage can in no wise be regarded as lending support to the belief that there is a causality of physical nature which is founded on rigorously exact laws. The extended field, "ether" is merely the *transmitter* of effects and is, of itself, powerless; it plays a part that is in no wise different from that which space with its rigid Euclidean metrical structure plays according to the old view; but now the rigid motionless character has become transformed into one which gently yields and adapts itself. ... (Weyl 1922, 311, emphasis in original)

Now the old duality of field ("ether") and matter was back again for our protagonist. This brought him closer to the perception of the problem by the majority of physicists working on the structure of matter and indicated a growing distance to the views held by A. Einstein.

A short-lived singularity theory making place to "agency structures" of matter

As Weyl came from as strong field theoretic paradigm, it was natural for him in the years 1920/21 to characterize matter by its formal relationship to the interaction field(s).[21] Thus in the fourth edition of RZM Weyl stated his new viewpoint clearly:

> Contrary to Mie's view, *matter* now appears *as a real singularity of the field*. (Weyl 1922, 262, emphasis in original)[22]

But then, matter had somehow to be located in a determinative boundary structure of the field and the old question of the structures of matter, which in the classical mechanistic view had been given by the assumptions of its atomic constitution and the hypothetical extension of mechanical laws to the atomic level, was again open. After the experience of dynamistic hopes during his period of adherence to the Mie theory, and in the light of recent modifications coming from experimental knowledge in microphysics, Weyl came to the conclusion:

> If matter is to be regarded as a boundary singularity of the field, our field-equations make assertions only about the *possible states of the field* and *not about the conditioning of the states of the field by the matter*. This gap is provisionally filled by the *quantum theory* in a manner of which the underlying principles are still

[21] For Weyl, his unification of the interaction still seemed valid, after his distachement from Mie's matter theory, and thus the singular form "field" would apply; scepticists with respect to his unification could easily reread his remarks by turning to the plural "fields".

[22] In the German original (Weyl 1918*b*, [4]1921, 238), no longer in the fifth edition.

> completely unmisunderstood. (Weyl 1922, 303, emphasis in original)[23]

Now the task to understand matter mathematically could be approached from different viewpoints. One was topological in nature. General relativity offered the opportunity to consider a differential topological manifold with boundaries, in the interior of which the fields are regular, while they are singular on the boundaries and diverge in respective limiting processes. In an article written for *Annalen der Physik* shortly after the publication of the fourth edition of RZM, Weyl explained his new viewpoint more in detail (Weyl 1921*a*). He argued in two directions. Coming from the point of view of special relativity and Minkowski space, the generalization for GRT consisted not only in a deformation of the metric but could also comprise a topological modification of the underlying manifold. Weyl argued that from the space-time manifold with a combined electromagnetic and gravitational field, the subsets on which the fields obtain singular values should be omitted.

> In the general theory of relativity the world can possess arbitrary (...) connectedness: nothing excludes the assumption that in its Analysis-Situs properties it behaves like a four-dimensional Euclidean continuum, from which different tubes of infinite length in one dimension are off. (Weyl 1921*a*, 252f.)

If the general relativistic point of view was considered as the more realistic one, it even appeared as more natural to turn the view round. One would then have to argue in terms of pasting rather than of cutting:

> The simply connected continuum from which we construct the domain of the field by cutting off the tubes is nothing but a mathematical fiction, although the metrical relations persisting in the field strongly propose the extension of the real space by addition of such fictitious improper (erdichteter uneigentlicher) regions corresponding to the single matter particles. (ibid.)

For Weyl, this change of the mathematical construction of space went in hand with a change of the understanding of the relationship between space-time and matter:

> According to [this] perception, *matter itself is nothing spatial (extensive) at all, although it is inserted in a certain spatial neighbourhood.* (Weyl 1921*a*, 254, emphasis in original)

He must have liked this idea. One of the Fichtean motifs on the "construction" of matter and space from "forces", which had impressed Weyl already

[23]The translation of the last phrase has been slightly changed to adapt it closer to the German original (Weyl 1918*b*, [4]1921, 276) than in Brose's translation. This passage is no longer contained in the fifth edition.

at the time of his turn towards "purely infinitesimal" geometry, ancquired here a new face and persisted in a modified form.[24]

On the other hand, there was a physical approach to the problem of matter. In addition to the proper laws of the field(s) one had to "study the laws according to which matter excites the field actions". For Weyl, matter was now turning into an irreducible originator of dynamical excitation of the interaction field(s) and was itself guided by the latter in its own spatio-temporal dynamics. He insisted that it could neither be understood as a "substance" in the sense of traditional natural philosophy, nor could it be derived from the "field" as in the Mie version of dynamicist matter explanation. Weyl preferred to give a description and a term of his own to characterize matter as an *agency (Agens)*. This word was uncommon in the German language, and, to my knowledge, even not used in the earlier discourses on natural philosophy. Weyl apparently shaped it on his own from the Latin gerundial form *agens* for something that is acting. In his usage of the word, an agency perception of matter was not far from the older dynamicist one of the philosophical debate of the early 19th century. As, however, the dynamistic view of matter had been linked to the electromagnetic world view and its generalizations in the Mie - Hilbert - Weyl approach by classical field theories, Weyl had good reasons to demarcate the break with this semantical field by the choice of a new word.

Different to the older field theoretical theories, Weyl considered it as an important feature of the agency view that it considered matter as something which acts on spatial structures like fields, although it is not itself located inside space. Already in 1921, several years before the advent of the refined form of the quantum mechanics, Weyl stated optimistically:

> In addition to the *substance* and the *field* perceptions we have to add a third view of matter as an *agency (Agens)* effecting the field states. . . . It makes place for the modern physics of matter, working with statistical concepts, besides the strictly functional physics of a classical field. (Weyl 1921*a*, 255)

For Weyl, such a shift had nothing to do with a longing for "acausality", or even the adoration of it. He rather insisted that the view of mechanical and classically field theoretic physics had reduced causality to a purely functional mathematical relationship, while the agency perception opened a possibility to understand the causation of field states by matter in a new and deeper way.

> Here the specified direction of the passage of time: past → future, which cannot find its place in field physics, can be taken up again; in fact it is most closely related to the idea of causation. (Weyl 1921*a*, 256)

[24] Compare (Scholz 1995, Scholz 2001*a*).

The characterization of causality by a deterministic and time-invertible law-like structure as in classical mechanics appeared him as an inappropriate concept. The change from classical determination to a probabilistic one would therefore not at all contradict the concept of "causation". Just to the contrary, Weyl expected that it might open the path towards a more appropriate understanding of the latter. Although his most recent turn had its origin in the short-lived singularity theory of matter, the "agency paradigm" of matter was kept open for a modification in its mathematical characterization and for a future enrichment by an improved understanding of its physical properties.

The agency concept of matter as an open research field

The role Weyl assigned to singularities of classical fields in the fourth editionoif RZM remained itself "singular" in his work. It did not appear earlier and vanished, or was at least drastically reduced in importance, nearly as fast as it appeared. In the fifth edition of his book the section on "further rigorous solutions of the statical problem of gravitation", which contained the central passages on the singularity theory of the electron, from which we quoted above, was completely reorganized. Apparently Weyl was not satisfied with the outlook on the strong interpretation of singularities as *the* mathematical clue to the solution of the "problem of matter". In the fifth edition and in his later publications on the philosophy of nature (Weyl 1924, Weyl 1927) we find the singularity model only in a weak sense, mentioned in passing as nothing more than an idea illuminating the impossibility to localize the basic agency structures of matter directly.

In the fifth edition of RZM, Weyl no longer gave the impression that he was already in possession of a mathematical clue to the solution of the "problem of matter". He now preferred to characterize only the terrain of investigation and discussed different approaches that had been tried, up to then. Among these he mentioned, of course, Mie's theory and his own generalization as important examples. But now they were only presented as explorative theoretical models, without any claim that they might lead towards a reliable representation of reality.

In this discussion we find beautiful, nearly poetic descriptions of the actual state of knowledge as an open terrain:

> We only percieve the bounding embankment of the subtle, deep groove which is dug into the metrical face of the world by the trajectory of the electron; what is covered by the depth, remains hidden to us. It may be that the whole groove is filled by a field, qualitatively equivalent to the outer one, as Mie assumed; *but just as well the abyss may be fathomless.* Mie's perception dissolves matter into the field; the other one removes it, so to

> speak, from the field. According to the latter view *matter is an agency determining the field, although in itself nothing spacelike, extensional, but only located in a certain spatial neighbourhood,* from which its field effects depart. ...
> (Weyl 1918*b*, ^{5}1923, 286)

Coming closer to the middle of the 1920s, Weyl left it open whether it seemed more promising to smooth off the field for a mathematical representation of the basic constituents of matter (like in the Mie approach), to excise it (like in the singularity approach of 1921), or to find any other characterization which might take the statistical nature of quantum descriptions better intor account than the other ones:

> Our description of the field surrounding an electron is a first, stuttering formulation of such laws. Here lies the working field for modern physics of matter, to which belong, above all, the facts and riddles of the quantum of action (...) As far as we can judge today, the lawfulness acording to which matter induces effects can be described in statistical terms only, ..." (RZM ^{5}1923, 286f.)

Independent of these open problems for an adequate mathematical characterization of matter, it now appeared clear to him that matter, rather than the field had to be given primacy for all experimental purposes or any practical exchange with nature.

> Our willful actions have, primarily, always to grapple on matter, only thus we can change the field. In fact, we then need two kinds of laws for the explanation of natural phenomena: 1. *field laws* (...), 2. *laws regulating the excitation of the field by matter.* .... (ibid.)

Finally causality came back, for Weyl, to be a relation which enabled human beings to influence the course of natural processes by a willful modification of material constellations in the world. As he had come to the insight that physical knowledge of the basic matter structures was still highly restricted, he cancelled those passages of the final sections of earlier editions of RZM, which appeared much too enthusiastic from his recently acquired view. That did not exclude poetic allusions. The fifth edition, the last one revised or extending by himself, ended with a passage which was both, sober and prophetic:

> We were unable to pursue our analysis of space and time without studying matter in detail. Here, however, we are still confronting riddles the solution of which is not to be expected from field physics. In the darkness still surrounding the problem of

matter, quantum theory may perhaps be the first twinkling of light. (Weyl 1918*b*, [5]1923, 317)

Weyl had entered the first phase of active intervention into mathematical physics (the "RZM-phase", as we might call it) with a strong program of reductionist unification; at the end of it, he clearly saw the necessity to distinguish ontologically and mathematically between interaction fields and matter. While for the first class the classical field theories could be considered as very successful, the problem of matter had turned back again into a riddle.

A view back in 1930

Only two years after these lines were written, the "first twinkling of light" was stabilized by the establishment of quantum mechanics in the form of wave mechanics and operator theory in Hilbert spaces. The core of this development was the product of a new generation of physicists (W. Heisenberg, W. Pauli, P. Jordan, P.A.M. Dirac, E. Schrödinger, e.a.) who stood in close communication with outstanding figures of the earlier period (N. Bohr, M. Born, A. Sommerfeld, P. Ehrenfest, e.a.). Although Weyl was no member of this group, he was close enough to several of the participants that he was immediately drawn into the turn to "the new" quantum theory at the middle of the 1920s. In oral and written exchange with E. Schrödinger, W. Pauli, M. Born and P. Jordan he even contributed in certain respects to it.[25] In his lecture course in winter semester 1927/28 on *Group Theory and Quantum Mechanics*, he took up Schrödinger wave functions and Pauli spinors (in later terminology) as new mathematical forms to represent a stochastically determining matter "agency". In the book arising from it (Weyl 1928) he could already integrate Dirac spinors for the characterization of a relativistic matter "field" of a new type. The second edition (1931) entered into the complex and irritating discussion of "second quantization" of these new provisional symbolic systems for the agency characterization of matter.

Knowing well about the provisional character of the quantum mechanical characterizations of matter, Weyl was deeply impressed by its successes already on the level of spectroscopy and the first steps into the quantum chemical theory of valence bonds. An invitation to give the 1930 Rouse Ball lecture at Cambridge gave Weyl the opportunity to review the whole development of matter concepts, which had taken place during the long decade just coming to an end.

Even from hindsight, he still considered the attempts of the early 1920s to geometrize "the whole of physics" as very comprehensible at its time, because they had tried to follow up on Einstein's successful geometrization of gravitation (Weyl 1931, 338). In this historizing perspective, he saw no reason to distance himself from his own attempts of 1918. He summarized

[25] For the Born and Jordan part see (Scholz 2004*b*).

its critical reception by physicists and reviewed Eddington's approach to unification by affine connections, including Einstein's later suppport for that program. Comparing the latter with his own "metrical" unification of 1918 he concluded that from hindsight both theory types appeared as "merely geometrical dressings (geometrische Einkleidungen) rather than as proper geometrical theories of electricity". He discussed the struggle between the metrical and affine field theories (i.e., Weyl 1918 versus Eddington/Einstein) and gave the whole story a smilingly ironic turn:

> ...there is no longer the question which of the two theories will prevail in life, but only whether the two have to be buried as twin brothers in the same grave or in two different graves. (Weyl 1931, 343)

In the light of his changed view on the problem of matter, he could find just as little arguments in favour of the more recent brands of unification attempts proposed at the end of the decade, Einstein's distant parallelism approach or the Kaluza-Klein approach.[26] Weyl completely rejected Einstein's new theory, not only by semantical reasons, but also by a mathematical one, because in his opinion it would break with the infinitesimal point of view, and warned:

> The result [of pursuing Einstein's *Fernparallelismus* approach, E.S.] is to give away nearly all what has been achieved in the transition from special to general relativity. The loss is not compensated by any concrete gain." (Weyl 1931, 343)

Weyl percieved a nearly complete scientific devaluation of all unified field theories invented during the long decade. This devaluation resulted from the quantum theoretical insights into matter structures, which had found first well formed mathematical representations by complex scalar or spinor fields during the second part of the decade:

> In my opinion the whole situation has changed during the last 4 or 5 years by the detection of the matter field. All these geometrical leaps (geometrische Luftsprünge) have been premature, we now return to the solid ground of physical facts. (Weyl 1931, 343)

He continued to sketch the theory of spinor fields and the new understanding ot the underdetermination of phase which opened a new theoretical frame for the gauge prinicple. In 1929 he and V. Fock had proposed a revised gauge theory of electromagnetism in this context. He insisted that the new principle of phase gauge "has grown from experience and resumes a huge treasury of experimental facts from spectroscopy" (ibid. 344). That stood in

[26]For Einstein's distant parallelism see (Sauer 2003), for Kaluza and Kaluza-Klein (Wuensch 2003).

marked contrast to the purely speculative principles on which all the classical unified field theories had been built, his own one from 1918 included. Now he no longer expected to achieve knowledge on natural processes by geometric speculation, but tried to anchor it in more solid grounds, the observation of matter processes and their mathematization:

> By the new gauge invariance the *electromagnetic field now becomes a necessary appendix of the matter field, as it had been attached to gravitation in the old theory.* (Weyl 1931, 345, emphasis in original)

In short, Weyl had turned from his speculative and strongly idealist approach to matter, pursued at the turn to the 1920s, *to a mathematically empiristic and moderately materialistic* one at the end of the decade. He was well aware that great difficulties had still to be surmounted to come to grips with a quantization of the semiclassical fields (complex scalar or spinor wave functions) which had recently been invented for a provisional and partial representation of the quantum properties of matter. That gave geometry a completely different outlook to the one in the classical field theories, although he did not want to exclude that geometrization might become possible some day on a new level. But if one wanted to continue along this path, he was sure that "one had to set out in search of a geometrization of the matter field" itself. If one would try to do without an improved mathematization of the agency structures of matter themselves, the geometrical theories would fall back to the methodological status of the unification attempts of the 1920s. He now considered these as immature, although comprehensible first attempts, as *Luftsprünge* (leaps into the air).

It may be appropriate to add that the German word "Luftsprünge" not only connotes unrealistic first attempts, but also the joy of youthfulness. Weyl has had both, the joy of the youthful speculation to be close to a reduction of physics to geometry, and the maturizing awareness that the difficult practices of experimentation and closely related mathematical theory production of quantum theory contained a much more reliable contribution towards the understanding of the "agency structures" of matter.

References

Brading, Katherine. 2002. "Which symmetry? Noether, Weyl, and conservation of electric charge." *Studies in the Philosophy of Modern Physics* 33:3–22.

Cao, Tian Yu. 1997. *Conceptual Developments of 20th Century Field Theories.* Cambridge: University Press.

Corry, Leo. 1999*a*. "David Hilbert between mechanical and electromagnetic reductionism (1910–1915)." *Archive for History of Exact Sciences* 53:489–527.

Corry, Leo. 1999*b*. "From Mie's electromagnetic theory of matter to Hilbert's unified foundations of physics." *Studies in the History and Philosophy of Modern Physics* 30:159–183.

Corry, Leo; Stachel, John; Renn Jürgen. 1997. "A belated decision in the Hilbert-Einstein priority dispute." *Science* 278:1270–1273.

Forman, Paul. 1971. "Weimar culture, causality, and quantum theory, 1918–1927: Adaptation by German physicists and mathematicians to a hostile intellectual environment." *Historical Studies in the Physical Sciences* 3:1–116.

Forman, Paul. 1980. "Kausaltät, Anschaulichkeit und Individualität, oder wie Wesen und Thesen, die der Quantenmechanik zugeschrieben, durch kulturelle Werte vorgeschrieben wurden." *Kölner Zeitschrift für Soziologie und Sozialpsychologie, Sonderheft* 22:393–406. In (von Meyenn 1994, 181–200).

Goenner, Hubert. 2004. "On the history of unified field theories." *Living Reviews in Relativity* . [http://relativity.livingreviews.org/Articles/lrr-2004-2, visited June 2, 2004].

Goldstein, Catherine; Ritter, Jim. 2000. "The varieties of unity: Sounding unified theories 1920–1930." Berlin: Preprint MPI für Wissenschaftsgeschichte149.

Gray, Jeremy (ed.). 1999. *The Symbolic Universe: Geometry and Physics 1890–1930.* Oxford: University Press.

Hendry, John. 1984. *The Bohr-Pauli Dialogue and the Creation of Quantum Mechanics.* Dordrecht: Reidel.

Hesseling, Dennis. 2003. *Gnomes in the Fog. The Reception of Brouwer's Intuitionism in the 1920s.* Basel: Birkhäuser.

Hilbert, David. 1992. *Natur und mathematisches Erkennen. Vorlesungen, gehalten 1919–1920 in Göttingen.* Nach Ausarbeitungen von P. Bernays. Hrsg. D. Rowe. Basel etc.: Birkhäuser.

Kohl, Gunter. 2002. "Relativität in der Schwebe: Die Rolle von Gustav Mie." Berlin: Preprint 209, MPI History of Science.

Letter H. Weyl to F. Klein, December 28, 1920. 1920. Nachlass F. Klein Universitätsbibliothek Göttingen Codex Ms Klein 12:297.

Pais, Abraham. 1986. *Inward Bound: Of Matter and Forces in the Physical World.* Oxford: Clarendon.

Renn, Jürgen; Stachel, John. 1999. "Hilbert's foundations of physics: From a theory of everything to a constituent of general relativity." Berlin: Preprint 118, MPI History of Science.

Rowe, David. 1999. "The Göttingen response to general relativity and Emmy Noether's theorems". *In (Gray 1999, 189–233).*

Sauer, Tilmann. 1999. "The relativity of discovery." *Archive for History of Exact Sciences* 53:529–575.

Sauer, Tilmann. 2003. "Field equations in teleparallel spacetime: Einstein's *Fernparallelismus* approach towards unified field theory." *Preprint Pasadena*, submitted to *Historia Mathematica.*

Schappacher, Norbert. 2003. "Politisches in der Mathematik: Versuch einer Spurensicherung." *Mathematische Semesterberichte* 50:1–27.

Scholz, Erhard. 1995. Hermann Weyl's "Purely Infinitesimal Geometry". In *Proceedings of the International Congress of Mathematicians, Zürich Switzerland 1994*. Basel etc.: Birkhäuser pp. 1592–1603.

Scholz, Erhard. 2000. Hermann Weyl on the concept of continuum. In *Proof Theory: History and Philosophical Significance*, ed. Vincent Hendricks; S.A. Pedersen; K. Frovin. Dordrecht: Kluwer pp. 195–220.

Scholz, Erhard. 2001*a*. "Philosophy as a Cultural Resource and Medium of Reflection for Hermann Weyl." *Preprint* Wuppertal to appear in *Révue de synthèse*.

Scholz, Erhard (ed.). 2001*b*. *Hermann Weyl's* Raum - Zeit - Materie *and a General Introduction to His Scientific Work*. Basel etc.: Birkhäuser.

Scholz, Erhard. 2004*a*. "Hermann Weyl's analysis of the "problem of space" and the origin of gauge structures." *Science in Context* 17:165–197.

Scholz, Erhard. 2004*b*. "The introduction of groups into quantum theory." *Preprint Wuppertal, submitted to* Historia Mathematica.

Sigurdsson, Skúli. 1991. "Hermann Weyl, Mathematics and Physics, 1900 – 1927." Cambridge, Mass.: PhD Dissertation, Harvard University.

Stöltzner, Michael. 2002. *Vienna Indeterminsm. Causality, Realism and the Two Strands of Boltzmann's Legacy (1896–1939)*. PhD Dissertation Bielefeld University.

Varadarajan, V.S. 2003. Vector bundles and conections in physics and mathematics: some historical remarks. In *A Tribute to C. S. Seshadri (Chennai, 2002)*, ed. V. Balaji, V.; Lakshmibai. Trends in Mathematics Basel etc.: Birkhäuser pp. 502–541.

Vizgin, Vladimir. 1994. *Unified Field Theories in the First Third of the 20th Century.* Translated from the Russian by J. B. Barbour. Basel etc.: Birkhäuser.

von Meyenn, Karl (Hrsg.). 1994. *Quantenmechanik und Weimarer Republik*. Braunschweig: Vieweg.

Weyl, Hermann. 1918*a*. *Das Kontinuum. Kritische Untersuchungen über die Grundlagen der Analysis*. Leipzig: Veit.

Weyl, Hermann. 1918*b*. *Raum, - Zeit - Materie.* Berlin etc.: Springer. Weitere Auflagen: ^{2}1919, ^{3}1919, ^{4}1921, ^{5}1923, ^{6}1970, ^{7}1988, ^{8}1993.

Weyl, Hermann. 1920. "Das Verhältnis der kausalen zur statistischen Betrachtungsweise in der Physik." *Schweizerische Medizinische Wochenschrift* 50:737–741. GA II, 113–122, [38].

Weyl, Hermann. 1921*a*. "Feld und Materie." *Annalen der Physik* 65:541–563. In (Weyl 1968, II, 237–259) [47].

Weyl, Hermann. 1921*b*. "Über die neue Grundlagenkrise der Mathematik." *Mathematische Zeitschrift* 10:39–79, GA II, 143–180, [41].

Weyl, Hermann. 1922. *Space, Time, Matter.* Translated from the 4th German edition by H. Brose. London: Methuen.

Weyl, Hermann. 1924. "Was ist Materie?" *Die Naturwissenschaften* 12:561–568, 585–593, 604–611. Reprint Berlin: Springer 1924. Darmstadt: Wissenschaftliche Buchgesellschaft 1977. In (Weyl 1968, II, 486–510) [66].

Weyl, Hermann. 1927. *Philosophie der Mathematik und Naturwissenschaft,* Handbuch der Philosophie, Abt. 2A. Weitere Auflagen ²1949, ³1966. English with comments and appendices (Weyl 1949). München: Oldenbourg.

Weyl, Hermann. 1928. *Gruppentheorie und Quantenmechanik.* Leipzig: Hirzel. ²1931, English 1931.

Weyl, Hermann. 1931. "Geometrie und Physik." *Die Naturwissenschaften* 19:49–58. Rouse Ball Lecture Cambridge, May 1930. GA III, 336–345 [93].

Weyl, Hermann. 1949. *Philosophy of Mathematics. and Natural Science.* 2nd ed. 1950. Princeton: University Press.

Weyl, Hermann. 1968. *Gesammelte Abhandlungen, 4 vols.* Ed. K. Chandrasekharan. Berlin etc.: Springer.

Wuensch, Daniela. 2003. "The fifth dimension: Theodor Kaluza's ground-breaking idea." *Annalen der Physik* 12:519–542.